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Real Structures and Divine Action Externalism: A Solution to Benacerraf's Problem

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Paul Benacerraf claimed in 1973 that realist accounts of truth which are applicable to propositions of both mathematics and science inevitably conflict with accounts of how propositions are known. This article presents a solution to the problem based on two assumptions: (1) that the God of the Bible is absolutely self-sufficient; and (2) that God is creating, providentially guiding, and redeeming according to his plans for his purposes in Christ. A coherent synthesis of what these assumptions entail itself entails that "abstract" and physical structures are real and that propositional knowledge of them is a matter of one's cognitive faculties functioning properly due to God's acting.

Keywords: Paul Benacerraf, truth, mathematical structuralism, structural realism, epistemic externalism, divine action, abstract objects, Jonathan Edwards, continuous creationism

I. Overview

In "one of the most influential articles in the last half century of philosophy of mathematics," Paul Benacerraf argues that realist accounts of truth, which are uniformly applicable to propositions of both mathematics and physical theory, inevitably conflict with accounts of how such propositions are known.[1] A *realist* account of truth presupposes that the world exists objectively and independently of the way one thinks about it and that a proposition is true in virtue of its corresponding, in some sense, to the way the real world is.

The source of the conflict lies in this. On the one hand, holding that we gain knowledge of physical objects by standing in a causal relation to them in virtue of our *perceptual* capacities, we have no explanation of how we can have mathematical knowledge, given that abstract objects are causally inert. On the other hand, holding that knowledge of mathematical truths is gained in virtue of our *conceptual* ability to detect self-evidence and the relation of logical consequence, we have no explanation of how we know propositions regarding empirical phenomena, given that they are not matters of conceptual self-evidence and *logical* consequence. Various interpretations of Benacerraf's problem have appeared in the literature along with proposed solutions and their criticisms. This vast literature indicates the pertinence of the problem to areas beyond the philosophy of mathematics to metaphysics and to the philosophy of science.[2]

As far as I am aware, none of the accounts of the concept of mathematical truth or accounts of propositional knowledge to which Benacerraf refers, nor any proposed solution, is built on Christian biblical theism. I propose a solution based on the ideas of Augustine of Hippo (354–430) and Jonathan Edwards (1703–1758), who held that the universe is a matter of God's purposeful, universe-sustaining action according to his plan in Christ.

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This feature alone makes this a "Christian" view. God's acting according to his plans for his purposes in Christ is the organizing principle of "History of Redemption" biblical theology.[3] It distinguishes Christian biblical theism from other versions of "theism."[4] God's plan is only one of a range of alternative histories for a universe, each of which God is aware in virtue of being eternally and perfectly aware of his power. Hence, what makes the propositional content of sentences of both physical theory and mathematics true—that, in virtue of which, true propositions are true—is ultimately the *way* in which God continuously creates according to his *plan*. Briefly stated, the solution may be stated in three stages.

First, *God's being absolutely self-sufficient* involves *God's being perfectly aware of his power and plans*. Since both God's perfect awareness of his power and God's plans or representations *for* creation are structured in several respects, there are *real "abstract" structures* (these are the truthmakers for mathematical propositions). On the surface of things, it might appear that *God's being absolutely self-sufficient* conflicts with *God's acting for purposes.*

Jonathan Edwards showed how these are consistent; this brings us to the second point. Edwards's reconciliation entails a metaphysics of creation—*idealism*, *continuous creationism,* and (physical) *occasionalism*—which, in turn, entails that there are *real physical structures* (these are the truthmakers for scientific and other empirical propositions).

The third stage is that God's creating according to his plan involves humans' perceiving, conceiving, and knowing propositions both of science and mathematics. Thus, neither abstract objects nor states of physical systems are the cause of our knowledge of them. Rather, God causes such knowledge by sustaining both our perceptual and conceptual capacities and the physical environment in which such capacities are situated and function. With this, we have an *externalist* account of propositional knowledge: a person's true belief is produced by a reliable belief-producing process of which they do not have exhaustive internal access.

The proposed solution here is more rigorous than stated above. It depends on the idea that *structures* are real and fundamental. Section II of this article describes two distinct ideas: (physical) *structural realism*, which is the core idea of a cluster of theories in the philosophy of science; and *realist mathematical structuralism*, which is the core idea of a cluster of theories in the philosophy of mathematics. It then describes two extant theories regarding their correlativity. Section III defines the propositional content of physical theory and mathematical theory in these structuralist terms.

Section IV provides a *theological* understanding of physical and mathematical structures, and of how they are ontologically connected, and shows of how these are logical consequences of the metaphysics of Augustine and Jonathan Edwards. Section V presents an "*occurrent content*" view of propositions derived from this metaphysics; this view stands in contrast to the standard "*platonic entity*" view held by most Christian philosophers.

Section VI presents a formal account of truth that is derived from this theological understanding of mathematical and physical structures. Section VII describes how the combination of these ideas can ground an *externalist* account of propositional knowledge. With all of this in hand, we have a rigorous solution to Benacerraf's problem based on biblical theism.

II. Structuralism: Mathematical and Physical

Mathematical structuralism is a philosophy of mathematics, standing as an alternative to *objects-platonism*, *logicism*, *formalism,* and *constructivism* regarding the nature of mathematical objects and how they are known. The idea was first proposed near the beginning of the twentieth century by Richard Dedekind (1888) and Henri Poincaré (1902).[5] The last quarter of the twentieth century saw the development of structuralist views of mathematics along several distinct lines, mirroring the debate between realists and antirealists over the status of abstract objects in general.[6] As a result, there are *platonist*, *aristotelian*, and *nominalist* views of mathematical structures. The prominent contemporary expressions of these competitors are Stewart Shapiro's (realist) *ante rem* structuralism, Michael Resnik's (realist) *in re* structuralism, and Geoffrey Hellman's (nominalist) *modal structuralism*, respectively.[7]

All realist versions of mathematical structuralism hold that mathematics should not be thought of as

the science of number and quantity, nor as a meaningless but useful formalism, nor as the study of a set-theoretic universe, not even as (merely) possible structures. Rather, the proper subject matter of mathematics is *real* structures. Accordingly, mathematical *objects* are "places" ("offices" or "positions") in structures, and their *identity* and *essence* are matters of the totality of relations they have to other places in the structure. Shapiro holds that mathematical structures and their places "exist independently of whether there are any systems of objects that exemplify them."[8]

Resnik describes realist mathematical structuralism as follows:

> In mathematics, I claim, we do not have objects with an "internal" composition arranged in structures, we only have structures. The objects of mathematics, that is, the entities which our mathematical constants and quantifiers denote, are structureless points or positions in structures. As positions in structures, they have no identity of features outside a structure.[9]

This article proposes a *realist modal structuralism* as an alternative to these views. Real structures are the *truthmakers* for the propositions of mathematics, but these are not Platonic abstract objects existing independently of God.[10] Rather, such real structures are aspects of God's representational awareness of his ability, whose primary element is God's plan in Christ. (This idea is developed in Section III.)

Physical structuralism holds that scientific theories do not inform us about the *nature* of what is modeled, but rather its structure. The most significant advocates may be James Ladyman and Don Ross, who hold *Ontic Structural Realism* (OSR), the view that the universe exhibits an ontologically fundamental, objective structure.[11] This article agrees with Ladyman and Ross on this point—that there are real physical structures. It differs from their theory primarily by holding that there *being* a complex structure to the universe and its components, and that structure's being *objective,* are matters solely of God's sustaining the universe *according to* his plan in Christ.

The literature regarding scientific and mathematical structuralism is extensive, and there are competing ideas regarding how real physical and mathematical structures are related at the metaphysical level. There is no need to review all of these and the particular issues that were at stake.[12] What is required for this article is only a characterization of the structural realism of this proposal. The next step toward that end is to properly situate what is being proposed within the context of contemporary philosophy of science and mathematics.

This article adopts *the structuralist conception* of scientific representation and *the mapping account* of the applicability of mathematics to physical theory. The structuralist conception of scientific representation is a development of the "semantic" view of scientific theorizing introduced by Patrick Suppes and Frederick Suppe.[13] Its aim is to describe how representation "works." Physical science represents real physical things of various sorts depending on the interests and purposes of the scientist and the phenomena they intend to individuate. The types of representations themselves vary accordingly. For example, a representation may be an equation, a drawing, a description, a probability distribution, a scale model, or something else. The structuralist conception holds that a scientific representation "works" in virtue of there being a similarity of *form*, *pattern*, or *structure* between its propositional content and its intended target object.[14]

The mapping account reflects the currently predominant consensus response to a problem famously raised by Eugene Wigner, the 1963 Nobel Laureate in Physics. Wigner posited that the

> enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it. The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.[15]

Wigner's paper provoked a plethora of responses from scientists, mathematicians, and philosophers, and the exact nature of the relation is far from settled. One sort of response, called "The Mapping Account," is the view that mathematics applies to the physical in virtue of the similarity (sometimes isomorphic) between the structures or patterns of the physical and a mathematical structure.[16] For both the structuralist conception and the mapping account, structures are fundamental. The task, now, is to specify and clarify the correlation and to provide examples.

III. Propositional Content of Physical and Mathematical Theory

Physical theories arise from perceptual experience, augmented in most cases by conceptual experience and imagination, and are often couched in mathematical terms. John Barrow observes that

> the intelligibility of the world amounts to the fact that we find [it] to be algorithmically compressible. We can replace sequences of facts and observational data by abbreviated statements which contain the same information content. These abbreviations we often call "laws of nature." This is why mathematics can work as a description of the physical world. It is the most expedient language that we have found in which to express those algorithmic compressions.[17]

There is perceptual/conceptual progression to our knowledge of the physical world. We perceive states of physical systems, then we abstract, idealize, and project detected *patterns* in repeated sequences of those states. We are further able to represent such patterns linguistically, thereby encapsulating extensive and often complex amounts of information. Examples are *Newton's Force Law*, F = ma; *Boyle's Law of Ideal Gases*, PV = cT; and most famously, *Einstein's mass/energy equivalence*, $E = mc^2$. What Barrow observes is helpfully expanded by Saunders Mac Lane:

> Mathematics begins with puzzles and problems dealing with combinatoric and symbolic aspects of the general human experience. Some of these aspects turn out to be systematic and intrinsic, rather than arbitrary and tied to one context. They become the stuff of elementary mathematics. From this starting point, the subject has developed to be a deductive analysis of a large number of very different but interlocking formal structures. *These structures have been derived from experience* [emphasis added] in many successive stages; by abstractions from various observations of the world, its problems, and the interconnections of these problems.[18]

The last sentence in the Mac Lane quote is salutary: "*These structures have been derived from experience*." It should be added that representation is intentional or purposeful and, at best, it approximates its target. Hence, *being similar to its target* is an insufficient characteristic of the relation of (structural) representation because the relation of (isomorphic) similarity is symmetrical, reflexive, and transitive, while representation is not. The difference lies in the intentional nature of a representation. As Bas van Fraassen observes: "*There is no representation except in the sense that some things are used, made, or taken, to represent things as thus and so* [emphasis given]."[19] Likewise, Otávio Bueno submits that a "representation is an *intentional* act relating two objects [emphasis given]."[20]

The point of this brief and admittedly selective discussion regarding the *origin* and *nature* of scientific representation is simply to underscore (1) the *dynamic context* within which mathematical and scientific propositions arise, (2) the nature of propositional representation as *approximate* at best, and (3) the *fundamentality of real structures*, thereby suggesting a way forward to a uniform theory of truth for both sorts of propositions in terms of divine purposeful action according to plan. Bearing in mind these features of the origin and nature of scientific representation, let us then proceed with this proposal:

> Let a *scientific representation* be the propositional content *p* of a set of sentences that is intended to represent either (1) a *state* of a particular physical system, detailing to some extent its *structure*, or (2) a *recurring pattern of states* (i.e., an apparent similarity) occurring either in one physical system at different times or in the states of several different physical systems (e.g., the geometric and bond structures of a sodium chloride crystal or *laws of coexistence*), (3) a *recurring pattern of change* in several states of one physical system or in the states of several different physical systems over some duration of time (e.g., a chemical and biological mechanism or *law of succession*).[21]

Let us turn our attention now to mathematics. There is a subtle, but crucial difference between a *mathematical theory* and a *mathematical structure*:

> A *mathematical theory* comprises two *sets* of (formal or informal) sentences and a (sound and complete) *system* of first-order logic. The *axiom* sentences express the essentials or lineaments of the subject matter of the theory and are taken for granted. The *theorem* sentences express the propositions which are derived from the axioms according to the system of logic. The system of logic comprises a *syntax*, a *semantics*, and a *deductive system*.[22]
>
> A *mathematical structure* is the complex concept (i.e., propositional content) expressed by a *mathematical theory* or by an *algorithmic compression*.

To avoid misunderstanding, it should be noted that the term "structure" is used in several other ways in

philosophy of science, philosophy of mathematics, and in mathematics itself. One prominent example is "algebraic structure," which consists of (1) a set of objects standing in defined relations arising from operations on that set and (2) a set of axioms which the relations and operations satisfy. Examples are groups, rings, fields, and lattices. In another sense, the term denotes a component in an axiomatic theory. In this latter sense, a structure is a list (i.e., a "tuple") consisting of a set of objects, an ordered sequence of relations (functions or operations) on the set, and names for distinguished elements in the set used for the interpretation of a formal language. Neither of these senses is intended by the definition. What I intend by use of the term "mathematical structure" is simply a common notion. Perhaps the best example is *Euclidean Geometry* as axiomatized by David Hilbert or Alfred Tarski.[23] The concept it expresses is *Euclidean space*. The "*relata*" of the structure are "*points*." Other examples are *Dedekind-Peano Arithmetic* (whose concept is an *omega sequence* whose relata, i.e., places or offices, are natural numbers), *Zermelo-Frankel* (ZF) set theory (whose concept is *the cumulative hierarchy of pure sets*), and *Causal Set Theory* (whose concept is *the discrete causal structure of spacetime*, whose relata are elementary events). Some mathematical structures such as the complex concept expressed by Causal Set Theory are also scientific representations.[24]

We have considered two types of propositional content, one associated with physical theory and the other with mathematical theory; both are structuralist. To serve the purposes of providing a solution to Benacerraf's problem, it must now be shown precisely how each sort of propositional content is related to divine action according to God's plan.

IV. An "Augustinian—Edwardsian" Synthesis

A *theological* understanding of physical structures and their correlativity to mathematical structures can be constructed from the metaphysics of Augustine and Jonathan Edwards. Both of them grounded their ideas in the biblical theme that G*od is creating, providentially guiding, and redeeming according to his plans for his purposes in Christ Jesus.* Augustine was the first to suggest that God's plan is only one of a range of alternative histories *for* a universe, things of which God is eternally aware in virtue of being aware of his power. Simo Knuuttila writes that God's

> acting by choice between alternative providential scenarios […] played an important role in the emergence of the intuitive idea of modality as referential multiplicity with respect to simultaneous alternatives. This modal paradigm hardly occurred at all among ancient thinkers. It was introduced in early medieval discussions which were strongly influenced by Augustine's philosophical theology.[25]

In short, the Augustinian element has two components: (1) God's plan in Christ is an ordered sequence of possibilities, only one among infinitely many alternative histories; and (2) God is perfectly aware of all possibilities in being aware of his power.

The second component is directly rooted in scripture and has been affirmed often in the history of Christian thought.[26] *That God is aware of his power and therein representationally aware of what is possible* is indicated most clearly perhaps by the rhetorical question posed several times in various settings, "Is anything too hard for the LORD?"[27] This pastorally motivated question is intended to evoke a response in the hearer. More is revealed implicitly. The posing of the question by God through the prophet presupposes that God is *aware* of his power. It could not have had its rhetorical effect had the hearer not been entitled to assume that God is aware of the answer. Edwards in his *Concerning the End for Which God Created the World* puts it this way:

> God as perfectly knew himself and his perfections, had as perfect an idea of the exercises and effects they were sufficient for, antecedently to any such actual operations of them, as since.[28]

What Edwards adds to the tradition is his showing how God's plan in Christ is a system of ends subordinate to God's "original ultimate end" and showing how this overcomes a famous conundrum.[29]

For Edwards and many others in the history of Christian thought, God's awareness of his ability *ad extra* is *representational*. These representations are not representations *of* things that exist, but representation *for* things to exist. God's *plan* in Christ is a history *for* a universe, according to which God confers existence, providentially guides the affairs of the world, and acts for his redemptive purposes. It includes a representation for every element of everything that falls under these categories, including an ultimate end in creation. It is like having a blueprint, a play, or a musical score in mind *prior* to the building of the house or *prior* to the performance.

The idea of *God's acting for purposes*, however, presents a conceptual problem which threatens the coherence of every metaphysical theory and systematic theology which holds it while also holding that *God is absolutely self-sufficient.* The ordinary concept of a person's acting to achieve an end entails three conditions *prior* to taking the first step in a course of action to achieve it, and *as* it is being pursued: (1) it is seen as having some objective value; (2) its achievement is actually treasured, cherished, or esteemed by the person pursuing it; and (3) something is gained by its achievement.[30] Hence, the concept of God's acting to achieve an ultimate end entails that God will gain something in achieving it and that God lacked it before creating, contradicting the idea that God is absolutely self-sufficient.

For centuries, this problem was well known among Jewish, Roman Catholic, and Protestant theologians. It was stated clearly by Baruch Spinoza in his *Ethics* (1677) and addressed in outline by Nicolas Malebranche in his *Dialogues* (1688). It was addressed in rigorous detail by Edwards in his *Dissertation Concerning the End for Which God Created the World* (1765). Today, it is almost completely unknown among contemporary Christian thinkers—at least inadequately appreciated and certainly seldom addressed.[31] Edwards was acutely aware of the conundrum and deductively demonstrates how the conjunction of the doctrines is conceptually coherent, thereby overcoming it.

Edwards's synthesis is crucial for this article in that it *logically entails* a version of physical and mathematical structural realism which is the conceptual foundation of the account of truth being proposed. To see this clearly, a brief explication of the problem and Edwards's solution is required. Edwards uses the phrase "absolute self-sufficience" three times, explicating it as follows: "God is infinitely, eternally, unchangeably, and independently glorious and happy."[32] *God's being absolutely self-sufficient* is revealed in God's names, *Yahweh* and *El Shaddai*. It is also revealed in scripture through prophetic statements, worshipful affirmations, and direct teaching. In eighteenth-century terms, Edwards observes that *God's being absolutely self-sufficient* includes at least three elements. First, God is *self-existent*. Second, God is *in himself excellent*, in goodness, in knowledge (wisdom), and in power. Third, God's Trinitarian nature is the sole source of God's dwelling in a state of complete *felicity and fulfillment*. Therefore, no created entity or set of conditions could be *necessary* for the *maintenance* of God's existence, excellence, or felicity or could be *sufficient* for the *reduction* or *increase* of any of them to any extent.

Accordingly, there are at least three concepts of value in accordance with which God might be thought to gain by creating something "external" to himself: (1) *ontological,* (2) *qualitative*, and (3) *psychological.* A thing's *ontological* value is its objective or real *existence*. A thing's *qualitative* value is its intrinsic *excellence*. Biblical cognates are "glory," "greatness," and "weightiness." The Hebrew word is *kabhodh*, דָּבָכ; the Greek (Septuagint) word is *dóxa*, *δόξα*. A thing's *psychological* value is the pleasure (felicity, joy, fulfillment) it provides. Again, for the sake of brevity, let's consider only the conceptual hypothesis of *ontological value* being added by God's creating. It alone is directly pertinent.

As did many theologians before him, Edwards recognizes that if God's ultimate end in creation involved the *addition* of real being, three contradictory positions follow. First, the sum total of all being would be greater than the being of God alone, thereby increasing God's glory or excellence by increasing his wealth and thereby increasing God's joy or fulfillment, which by supposition are unsurpassable. Second, the hypothesis contradicts the idea that creation is *ex nihilo*. Finally, if, at any moment of the additional being's existence, it were not completely dependent on God's *willing* its existence, then by definition it would be *real* with respect to God.[33] To exist independent of God's willing is to be self-existent. As Geerhardus Vos writes, "If it existed of itself, then so far as its being is concerned, it would be like God."[34] Here we have the only extant coherent account of how *God's acting for purposes* is consistent with *God's being absolutely self-sufficient.* It entails that the existence of the universe and all that belongs to it, at every moment of its existence, be *ideal* with respect to God. The universe in every respect depends radically and entirely on God's willing its nature and existence. If it were not, it would at that moment and in that respect be *self-existent*, which is a property only God can have.

Traditional systematic theology holds that *aseity* (self-existence) *is incommunicable.* Moreover, if the universe were not radically dependent on God's willing its nature and existence, God would gain "wealth" by creating. This idealism that is a logical

consequence of Edwards's synthesis is a position regarding *ontological dependence*; it is not an idealism of physical *fundamentality* or *constitution*.[35] The ideas that God had perfectly in mind before creation as *representational-intentions-for-existence* become ideas as *divine-acts-of-willing-existence* according to his plan—two kinds of divine ideas, but ideas in God's mind, nonetheless. In other words, there is a change in the ontological *form* of God's ideas regarding creation, but no additional being is added, because creation is *ex nihilo*.[36] (This sort of idealism makes sense of interpretations of Quantum Field Theory and Quantum Gravity which hold that what is fundamental are not "bits of matter," but elementary events which are excitations of quantum fields.[37])

To reiterate, the existence and nature of the universe depend entirely on God's continuously willing it according to his plan. Hence, physical systems cannot have "ontological momentum" or "existential inertia." Moreover, the apparent causal relation between events or between states of physical systems lies solely in the sequence of God's existence-conferring action according to his plan. In other words, Edwards's development of the Augustinian ideas in overcoming Spinoza's conundrum also entails *continuous creationism* and (physical) *occasionalism*.[38] These two views are essential to the proposed solution to the Benacerraf Problem presented in this article and are entailed by Christian biblical theism.

The crucial point of this extended discussion is this: *God's being absolutely self-sufficient, God's being perfectly aware of power,* and *God's acting according to his plan for his purposes in Christ* together *entail* structural realism in three respects. First, God's awareness of his ability *ad extra* is structured in the form of alternative histories *for* a universe. Second, God's plan in Christ is structured. God's plan is real with respect to creatures and is a *system* of ends, which are events and stages in the history of redemption. By analysis of the concept of *system*, God's plan involves structure. From another angle, as a *history for a universe*, God's plan is an ordered sequence of component representations *for* divine action. It must have a *proto-temporal* structure, which is a linear order of "places" whose contents are these component representations. Third, the universe itself existing solely as a matter of God's creating *according to* his plan in Christ is also structured. God's plan—being a history for a universe and a system of ends—must also have a *representational* structure, which includes all of the relations between and among these components. Part of this structure must be a *proto-causal* structure, which is the *abstract* structure of those relations between and among these component representations, which becomes the *causal* structure of the universe as God continuously creates according to plan. Dispositions, laws of nature, chemical bond structures, and biological mechanisms are the manifestations of God's acting.

In short, since God is perfectly aware of his power and since God's plans or representations *for* creation are structured in several respects, there are *real* "*abstract*" *structures* (these are the truthmakers for mathematical propositions) and, given the metaphysics of creation (*idealism*, *continuous creationism,* and [physical] *occasionalism*), there are *real physical structures* (these are the truthmakers for scientific and other empirical propositions). Section VI is devoted to showing how these play a role in truth and knowledge. To make the case in that section, we must first describe the view of *propositions* assumed in this article that contrasts with the divine *plans*, which are God's representations *for* existence.

V. Propositions Distinguished from Divine Representations

A *proposition* is the informational (or information-like) *content* of an occurrent intentional mental state of a created agent. It is a short-lived, abbreviated, synoptic representation of some state of affairs or concept which is introspectively accessible and has a phenomenology of being necessary and abstract.[39] This "*occurrent content*" view of propositions, grounded in God's acting, stands in contrast to the standard "*platonic entity*" view held by most Christian philosophers. As Friederike Moltmann has observed,

> A number of philosophers have therefore argued that the notion of an abstract proposition, conceived as a formal object of one sort or another, should be replaced by a cognitive notion of proposition, a mind-dependent object whose truth-directedness is tied to the intentionality of an agent.[40]

To reiterate what was stated earlier, God is perfectly aware of his power, which is his ability *ad extra.* God's awareness is *representational*. These representations are not representations *of* things that exist, but representations *for* things to exist. God's *plan* in Christ is a history *for* a universe, according to which God confers existence, providentially guides the affairs of

the world, and acts for his redemptive purposes. It includes a representation for every element of everything that falls under these categories, including an ultimate end in creation. The point is that *God's representations for existence* are not propositions. Even though both are representations in a generic sense, they are distinct. Propositions can only be gross abbreviations of the content of divine thoughts, which are themselves exhaustively detailed representations *for* existence.

This difference affects philosophy of mathematics. Recent Christian philosophy of mathematics has taken the form of either *theistic activism*[41] or *theistic conceptualism*[42] regarding the existence and nature of abstract objects in general, while ignoring or denying the difference between propositions and God's representations. *Theistic conceptualism* takes the referents of mathematical terms (and other abstract objects) to be *concepts in* the divine intellect; *theistic activism* takes them to be *products of* the divine intellect. My view has both conceptualist and activist features, but it differs from them in that it treats abstract objects as a matter of God's representational *awareness* of his ability to create; they are representations *for* existence. Hence, God's "thoughts" cannot be propositions.

Conceptualizing God's plan in Christ as a sequence of divine representations for existence, and not as some sort of "possible world" or "book on a world" comprising propositions, precludes the threat of paradox in several ways.[43] First, since it is a composite plan, it must be a *sequential* and *infinite* representation *for* the universe. The possibility of global self-reference is eliminated because, unlike propositions, divine representations *for* existence cannot be self-referential. Second, since representations as plans are not self-referential, no power set of representations is entailed. Third, since God's plan in Christ is a complex relational structure whose parts are themselves mereological sums, each component plan that is not a simple representation is a *convex* sequence of such. This also precludes the construction of a power set. For these reasons, this structure is not subject to *Russell's Paradox*. For the same reasons, it is not subject to incoherence by applying *Cantor's Theorem*. In today's language, letting *the power set* $\wp(A)$ of a given set *A* be the set of all the subsets of *A*, *Cantor's Theorem* holds that, for any set *A*, *A* has fewer members than its power set $\wp(A)$. In particular, Cantor showed by way of an indirect *diagonal argument* that, for any set A, there is no one-to-one function from $\wp(A)$ into A. But again, since God's representational awareness of his ability *ad extra* is a mereological whole whose parts are convex, there is no power set of the collection of all of God's representations.

VI. A Formal Account of Truth

This Augustinian-Edwardsian development of Christian biblical theism suggests how to state a formal account of truth in structuralist terms. The first step toward that statement is to describe its general approach to *propositional truth*. William Alston proposes a "realist conception of truth," which he offers as an "account of our ordinary concept of truth."[44] Alston says that it is a "way of thinking of truth in that the truthmaker is something that is objective vis-à-vis the truthbearer and has to do with the relation of a potential truthbearer to a REALITY beyond itself."[45] This affirms Andrew Ushenko's claim that "semantics cannot be confined to purely linguistic relationships because it contains such concepts as 'truth,' which in the sense of 'agreement with reality' involves reference beyond discourse."[46] The same idea is voiced by Donald Davidson observing that relativized notions of truth in model-theoretic semantics—*truth-in-a-model* (i.e., *truth-under-an-interpretation*)—fail to express our ordinary nonrelative concept of truth.[47]

What is needed is a formal semantics that indicates *how* a proposition *p* which is *true-in-a model* is true *per se* by giving its truth-conditions in terms of *reality as it is*. Reality *as it is*, however—*reality beyond* every truthbearer—is a matter of God's purposeful, universe-sustaining *action according to his plan*, which is only one of a range of alternative histories *for* a universe, things of which God is eternally aware in virtue of being aware of his power. Hence, mathematics and physical theory are ultimately about *what God knows*, *how God acts in creating,* and *what exists as purposeful and planned divine creation*. With this in hand, we may now address the following question:

> Under what necessary conditions are *scientific representations* true *per se*, given that they can be only approximations?

Given the assumed theological metaphysics, a proposition of physical theory or of mathematics is true *per se* if (1) its *ultimate* referent—its *truthmaker*—is an element of *what God knows* regarding his ability *ad extra,* his purposes, and his plans and (2) its *intended*

referent is *what exists* as God's conferring existence to the universe.

To reiterate, God is perfectly and completely aware of God's power. God's awareness of God's power is in the form of an array of alternative histories for a universe, each of which is an ordered sequence of representations for divine action. For the sake of the precision of formalization, let $\mathcal{K}$ denote God's knowledge of power. Medieval theologians and philosophers used the expression, *potentia Dei absoluta*, which means "the absolute power of God." God's power is God's competence to bring about various states of affairs ad extra (i.e., outside of himself) as they are guided by his wisdom within the constraints of his holiness. It is God's ability to create, to providentially guide, and to redeem. Let us henceforth use the expression "God's ability ad extra" to refer to what we colloquially refer to as "God's power."

Let the italicized letter *r* be a (divine) representation *for* the result of a creative act. Hence, $\mathcal{K}$ includes *r*. A scientific representation, by contrast, is a human idea; it is a proposition which can only approximate what it denotes. Hence, let the expression consisting of the lower-case letter in brackets [*p*] be the element *r* of $\mathcal{K}$ represented by the propositional content *p*. In other words, the expressions '[*p*]' and '*r*' denote the same thing. The difference between the two expressions lies only in the *sense* of what they denote. The sense of the expression '*r*' is its being a divine *representation for* something's existence. The expression '[*p*]' denotes the exhaustively detailed component of God's representational awareness of his ability *ad extra*, insofar as it is ultimately represented by a proposition *p*. Because the existence of a physical system *x* is a matter of God's conferring existence to *x* according to his plan and since *r* is a constituent of God's plan according to which he confers existence, we can say: "*p* represents *r*" even though seldom (if ever) is a created agent conscious that *this* is what *p* ultimately represents. Since a true proposition *p* is—and can only be—an abbreviation of *r*, God's knowledge $\mathcal{K}$ includes [*p*]. Hence, in general, for any extant proposition *p* regarding a physical phenomenon or state of a physical system,

> *p is true per se* only if what *p* represents as being the case is included in God's knowledge $\mathcal{K}$.

This is the general form, though it is far from complete. To render this more precise, consider that since the universe is a matter of God's conferring existence according to his plan in Christ, the *truthmakers* for propositions regarding the physical world (i.e., scientific representations) are located in God's plan for creation. A truthmaker's "*actuality*" is prior to human ideas and to the existence of the entire universe itself (which is ideal and derivative). God's plan *for* the universe is *eternal* and *necessary* (being an aspect of God's eternal and necessary self-awareness). Since it encompasses all that God does and according to which every created thing owes its existence, nature, and functioning, God's plan alone deserves the name, "the actual history, α."

With this in mind, the truthmaker [*p*] lies in God's plan for the universe—the actual history α. It is either an *aspect* of α or "*located within*" α over some duration δ, where δ is a sequence of units *for* a minimal moment of existence (i.e., "frames for time"), not *of* moments of physical time itself. Furthermore, by definition, the truthmaker [*p*] for a scientific representation *p* is a real *possibility*. In other words, the truthmaker [*p*] is a *representation for* a *result* of an existence-conferring *act*. As discussed above, Augustine, Edwards, and scripture affirm that God, being perfectly aware of his ability ad extra, is perfectly aware of all real possibilities, as something he can create. With these refinements in hand, we have this more precise account:

> *p is true per se* only if God's knowledge $\mathcal{K}$ includes a representation *r* which is proposition *p*'s ultimate referent [*p*] and, for some duration δ, the actual history α includes [*p*] over duration δ.

We have stated two necessary conditions. Two more must be included to complete this account. The third is some created agent thinking a thought with proposition *p* as content. No proposition is true without this. Given the metaphysics of creation thus far derived, a *proposition* is the informational content of the *manifestation* of a mental disposition to believe. It is a person's subjective *intension* upon understanding a declarative sentence, existing briefly as the informational aspect of an occurrent intentional mental state. Hence, a proposition is an aspect of an event. Let the following, then, be the third condition:

> *p's being the content of some created agent's occurrent propositional attitude over δ is itself a component plan included in the actual history.*

An agent thinking a thought—just as the universe and every physical system it comprises—is a *process* of divine action according to God's plan. (This

element is crucial to the externalist epistemology to be described in section VII.) Thus, the fourth necessary condition is *God's existence-conferring action*. Without either of the latter two conditions, no proposition exists, much less has the property of *being true*. Accordingly, we have this refinement:

THESIS ONE:

> For any *scientific representation p*, *p is true per se* if and only if (1) God's knowledge $\mathcal{K}$ includes a representation *r* which is *p*'s ultimate referent (that is, $r = [p]$), such that (2) for some duration δ, the actual history α includes $[p]$ over δ, (3) *p's being the content of some created agent's occurrent propositional attitude over δ is itself a component plan included in the actual history*, and (4) *God is conferring existence according to his plan for his purposes in Christ*.

(This account expresses and develops the *correspondence conception of truth* understood in an ontologically realist sense.)

We have been addressing this question: Under what necessary conditions are scientific representations true, given that they can only be approximations? This stage in the development of the account takes one step, *reflecting* a grounding of the truth of a proposition in God's knowledge and existence-conferring action, where the proposition is of physical theory. None of the mathematical sentences mentioned above is a typical sentence of mathematics and its typical objects. Since what we require is a semantics regarding *truth per se* which applies coherently and unequivocally to propositions and theories of *both* physical theory and mathematics, the formal semantics must (somehow) apply *also* to the latter. The issue is how these two sorts of propositional content may be related to each other and then to *truth per se*.

Perhaps someone will now warn or protest that pure mathematics is the study of structures independently of whether they are exemplified in the physical universe. Accepting the thesis of the objection for the sake of argument, how are we to understand the nature of such structures? Are they constructs of the imagination only? Works of fiction are analogous examples. If so, what would it even mean to say that they are true per se? Just because we *can conceive* of some particular set of objects and some particular relations between those objects does not mean that those things are real.[48] (This issue is addressed below by Thesis Three.) With these definitions and examples in hand, we can address these questions:

> Under what necessary conditions are *mathematical structures* true *per se*, given that they can only be approximations?
>
> What sorts of things are *their* truthmakers—as the intended referents and the ultimate referents of such structures?

Real structures

Saunders Mac Lane reports that "mathematics consists in the discovery of successive stages of the structures underlying the world with emphasis on those structures of broad applicability and those reflecting deeper aspects of the world."[49] Similarly, *Ontic Structural Realism* (OSR) in the philosophy of science is the view that scientific theories do not inform us about the *nature* of what is modeled, but rather its *structure*.[50] The "structures underlying the world" to which Mac Lane and OSR refer are *real*. By definition, "*real structures*" do not depend on a human conceptualization for their existence. Therefore, the ultimate referent of a *true mathematical* structure is a *real* structure. (This satisfies *the mapping argument*). Here then, we have an *initial* suggestion of how to think about the truth conditions of propositions of mathematics.

As briefly described above, (1) *God's plan in Christ is an ordered sequence of possibilities, only one among infinitely many alternative histories* and (2) *God is perfectly aware of all possibilities in being aware of his power.* There are at least two broad types of *real* structures: Type I, the structures of *God's knowledge ad extra;* and Type II, the structures of *what we* perceive in perceiving the *result* of such acts—structures of, between, and among simple and complex physical systems.

For the sake of clarity, an elaboration is in order. A Type I real structure is a component of God's representational awareness of his ability *ad extra*. As such, it is either the structure of the array of alternative histories *for a universe,* or a structure of the actual history α (the history for our universe) as a system of ends, or a structure of a component of α. For example, God's plan *for* any two or more objects standing in some relation, involves a representation *for that relation*. A Type II real structure is the *result* of a way God confers existence according to the actual history α, God's plan in Christ. A representation *for* a Type II real structure (say *for* a sodium

chloride crystal or the mechanism of plant germination) is a complex representation: a representation which is itself a *relation* of representations for states of a physical system (occurring simultaneously in the sodium chloride example or sequentially in the mechanism example). In short, some representations are Type I real structures. Some of these are *for physical* structures, such that when enacted are Type II real structures. Thus, Type II real structures depend on Type I real structures.

Mathematical structure in relation to real structure

To reiterate, a mathematical structure *p* is true *per se* if and only if its ultimate referent is a divine representation. This general statement can now be developed further for the purpose of stating an account of *truth per se.* The type of structures to which Mac Lane and OSR refer are all physical structures. To reiterate, a physical structure is the continuing *result* of one of the continuing *ways* God confers existence.[51] For example, Meinard Kuhlmann observes that ontic structural realism takes

> symmetry groups to indicate that symmetry structures as such have an ontological primacy over objects. However, it is not altogether clear *how symmetry structures could be ontologically prior to objects* if they only exist in concrete realizations, namely in those objects that exhibit these symmetries.[52]

If what I am proposing is true, symmetry structures are Type II real structures ultimately grounded in one of God's representations *r*, which is a Type I real structure.

A true mathematical structure, in other words, involves a "chain" of referents terminating at its truthmaker, which is an element *r* of God's representational awareness of his ability *ad extra*, where *r* is a Type I real structure included in the actual history α. In other words, a true *mathematical structure*, which is also a *scientific representation,* correlates first to a Type II real structure and ultimately to a Type I real structure of the actual history α. Some true mathematical structures, however, are not scientific representations, but they correlate directly and only to a Type I real structure of the actual history α. The referent of a true mathematical structure is simply one of these real structures. The implications of these for a theory of truth for mathematical propositional content *p* is this account:

THESIS TWO:

> A *mathematical structure p* is *true per se* if and only if (1) God's knowledge $\mathcal{K}$ includes a representation *r* such that *r* is *p*'s ultimate referent (that is, *r* = [*p*]), such that (2) *r* is a real structure, where for some duration δ the actual history α includes [*p*] over δ, (3) *p's being the content of some created agent's occurrent propositional attitude over δ is itself a component plan included in the actual history*, and (4) *God is conferring existence according to his plan for his purposes in Christ*.

A *theorem* of a mathematical theory which is also intended to be a scientific representation of an *aspect* or a *component* of a physical system is *true per se* if and only if the actual history α includes a representation for it.[53] Hence, we have this corollary:

COROLLARY:

> A *mathematical proposition p* (if not intended as a scientific representation) is *true per se* if and only if there is a *mathematical theory* whose correlative mathematical structure is *true per se* and *p* is a theorem of that theory.

In other words, a proposition *p* of mathematics is *true per se* only if *p* is a logical consequence of a *true per se* mathematical structure.

Consider the following example, where the expression "<1+1=2>" denotes the proposition expressed by the mathematical sentence "1+1=2," the expression "Seq^{ω}" ("omega sequence") denotes an infinite sequence, the expression "$\wedge PA^2$" denotes the conjunction of the axioms of second-order Peano Arithmetic, and "α" denotes God's plan in Christ. Here is an analysis of the truth conditions of the proposition:

> The proposition <1+1=2> is true if and only if (1) God's representational awareness $\mathcal{K}$ includes a representation *r* for the omega sequence Seq^{ω} expressed by second-order Peano Arithmetic ($\wedge PA^2$), where (2) Seq^{ω} is a structure included in α and *1+1=2* is a theorem of $\wedge PA^2$, (3) (P) *p's being the content of some created agent's occurrent propositional attitude over δ is itself a component plan included in the actual world*, and (4) (G) *God is conferring existence according to his plan for his purposes in Christ.*

The *corollary* satisfies Benacerraf's requirement that "any theory that proffers theoremhood as a condition of truth also *explain the connection between truth*

and theoremhood."[54] In the *corollary*, theoremhood—that is, *being a theorem* of a mathematical theory—is indeed a condition of *p*'s being true *per se.* The "connection" is this: the truth of the proposition is "*transferred to it*" (so to speak) from the truth of the mathematical structure in virtue of the relation of logical consequence.

THESIS THREE:

> A *mathematical structure or theorem p* is *fictionally false* if and only if (1) God's knowledge $\mathcal{K}$ includes a representation *r* such that *r* is *p*'s ultimate referent (that is, $r = [p]$), and (2) *r* is a real structure but not included in the actual history α, and (3) *p's being the content of some created agent's occurrent propositional attitude over δ is itself a component plan included in the actual history*, and (4) *God is conferring existence according to his plan for his purposes in Christ*.

A fictionally false *mathematical structure p* is conceptually possible because it is logically consistent.[55] If a mathematical structure *p* is neither *true per se* nor *fictionally false*, it is *purely false*. God's knowledge $\mathcal{K}$ does not include a representation *r* such that *r* is *p*'s ultimate referent (that is, $r = [p]$).

In these three theses we have a formal account of truth which (1) is uniformly applicable to propositions of both physical theory and pure mathematics, (2) is derived from the fundamental reality that *God is creating, providentially guiding, and redeeming according to his plans for his purposes in Christ Jesus*, and (3) is sufficiently rigorous so as to serve a formal semantics of a system of quantified modal logic.[56]

VII. A Solution to Benacerraf's Problem

Stewart Shapiro observes that "most contemporary philosophy of mathematics begins" with Paul Benacerraf's argument that attempting to address two reasonable concerns leads inevitably to an impasse.[57]

Benacerraf's concerns are

> (1) for having a homogeneous semantical theory in which semantics for the propositions of mathematics parallel the semantics for the rest of the language, and (2) that the account of mathematical truth mesh with a reasonable epistemology.[58]

He concludes that "accounts of truth that treat mathematical and nonmathematical discourse in relevantly similar ways do so at the cost of leaving unintelligible how we can have any mathematical knowledge whatsoever."[59]

As it stands, Benacerraf's argument is not entirely obvious. Since this article is concerned with a uniformly applicable *realist* account of truth, it will be helpful to consider Shapiro's suggestion that the problem is a dilemma. A dilemma begins with a choice between at least two alternatives, whose consequences conflict. Shapiro approaches Benacerraf's problem as grounded in a choice between a realist and antirealist ontology regarding the objects of mathematical and physical theory.[60] Each alternative entails a problematic result. Only the realist alternative is pertinent.

This article proposes a uniformly applicable account of truth *per se* on the assumption that the propositional content of sentences of *both* physical theory and mathematics is ultimately about God's knowledge *ad extra* and about the ways God continuously creates according to his plan. Divine continuous creation pertains also to humans' perceiving, conceiving, and knowing; these three require believing a proposition. A created agent's *believing* a proposition is, in one sense, (1) an occurrent, intentional, doxastic state of consciousness and, in another sense, (2) a disposition to be in such states.

Given the divine action metaphysics presented earlier, even though the formation of choices of agents may not be produced by God-given free will, states of consciousness are produced by God. The *process* of coming to have the belief is inaccessible to the knower's consciousness. Hence, this is an instance of *externalism* with respect to *warrant*, where *warrant* is the element that, when added to justified, true belief, gives us propositional knowledge.[61] In addition and by contrast, the *justification* of an agent believing in *p* is (at least in part) grounded in what is *internal* to the mind, that is, the person is aware of reasons to think that *p* is true. The following statement succinctly combines *externalism* with respect to *warrant* with *internalism* with respect to *justification*:

> For any proposition *p* and any person *S*, *S knows that p* if and only if (1) *p* is true, (2) *S* believes that *p*, (3) *S* is aware of reasons for *p*'s truth and (4) both *S's* believing that *p* and *S*'s awareness of supporting reasons—the intentional state of consciousness and the propositional content in both cases—are produced by God even though *S*'s attending to *p* need not be.

This is sufficient because Benacerraf requires *only* that an account of propositional knowledge be *conceptually possible*—how one *might* have mathematical knowledge. As he puts it,

> an account of mathematical truth, to be acceptable, must be consistent with the possibility of having mathematical knowledge: the conditions of the truth of mathematical propositions cannot make it impossible for us to know that they are satisfied. To put it more strongly, the concept of mathematical truth, as explicated, must fit into an over-all account of knowledge in a way that makes it intelligible how we have the mathematical knowledge that we have.[62]

The account of truth *per se* combined with this externalist epistemology coherently satisfies both of the concerns noted by Benacerraf, thereby providing a solution.

A summary of the core idea of this article is this. Jonathan Edwards held with Augustine that God's plan in Christ is one of infinitely many alternative histories *for* a universe. On biblical grounds, Edwards provides a description of how *God's being absolutely self-sufficient* is consistent with *God's acting for purposes in Christ*. This coherent synthesis entails *idealism, continuous creationism,* and (physical) *occasionalism,* which ground real *physical* structures according to the real *abstract* structures of God's plan in Christ. A proposition of science or mathematics is true just in case its truthmaker is an element of one of these structures, and is known because God produces the belief according to his commitment to create, providentially guide, and redeem according to his plan in Christ. This solution to Benacerraf's problem is also an apologetic argument *for* Christian biblical theism.[63]

Notes

[1]See Bob Hale and Crispin Wright, "Benacerraf's Dilemma Revisited," *European Journal of Philosophy* 10, no. 1 (2002): 101, https://doi.org/10.1111/1468-0378.00151; and Paul Benacerraf, "Mathematical Truth," *Journal of Philosophy* 70, no. 19 (1973): 661–79, https://doi.org/10.2307/2025075.

[2]See Adam Morton and Stephen P. Stitch, eds., *Benacerraf and His Critics* (Hoboken, NJ: Blackwell Publishers, 1996); Fabrice Pataut, ed., *Truth, Objects, Infinity: New Perspectives on the Philosophy of Paul Benacerraf* (Switzerland: Springer Verlag, 2017); and Majid Davoody Beni, "The Benacerraf Problem as a Challenge for Ontic Structural Realism," *Philosophia Mathematica* 28, no. 1 (2019): 35–59, https://doi.org/10.1093/philmat/nkz022.

[3]See Edward W. Klink III and Darian R. Lockett, *Understanding Biblical Theology: A Comparison of Theory and Practice* (Grand Rapids, MI: Zondervan, 2012), 59–90.

[4]"Theism" is the belief that there is at least one god. As such, the term applies to the three Abrahamic religions: Judaism, Christianity, and Islam. The term also applies to the philosophical expressions of these religions such as (Aristotelian) classical theism and Anselmian theism. It also applies to systems such as Whiteheadian process theism and, of course, other sorts of monotheism.

[5]Richard Dedekind, *Was sind und was sollen die Zahlen?* [1888], trans. Wooster Woodruff Beman as "The Nature and Meaning of Numbers," in Dedekind, *Essays on the Theory of Numbers* (Chicago, IL: Open Court, 1901; reprint, Mineola, NY: Dover Publications, 1963): 31–115; and Henri Poincaré, *Science and Hypothesis*, trans. George Bruce Halsted (1902; reprint, Mineola, NY: Dover, 1952).

[6]See Erich Reck and Georg Schiemer, "Structuralism in the Philosophy of Mathematics," in *The Stanford Encyclopedia of Philosophy,* Spring 2020 Edition, ed. Edward N. Zalta, https://plato.stanford.edu/archives/spr2020/entries/structuralism-mathematics/.

[7]Stewart Shapiro, *Philosophy of Mathematics* (Oxford, UK: Oxford University Press, 1997); Michael D. Resnik, *Mathematics as a Science of Patterns* (Oxford, UK: Oxford University Press, 1997); and Geoffrey Hellman, *Mathematics without Numbers*: *Towards a Modal-Structural Interpretation* (Oxford, UK: Oxford University Press, 1989). See also the *Oxford Handbook of Philosophy of Mathematics and Logic*, ed. Stewart Shapiro (Oxford, UK: Oxford University Press, 2007).

[8]Stewart Shapiro, *Philosophy of Mathematics: Structure and Ontology* (Oxford, UK: Oxford University Press, 2000), 9, 75.

[9]Michael D. Resnik, "Mathematics as a Science of Patterns: Ontology and Reference," *Noûs* 15, no. 4 (1981): 530, https://doi.org/10.2307/2214851.

[10]The version of Platonism—Platonism with a capital "P"—that holds that there is a realm of abstract objects existing independently of God is to be rejected. My proposal is a "platonism" with a lower-case "p," following a long tradition of denying Platonism and holding that what we take to be abstract objects (propositions, universals, possible worlds, and sets) are matter of God's thoughts, which I specify as God's thoughts regarding his power.

[11]James Ladyman and Don Ross, *Every Thing Must Go: Metaphysics Naturalized* (Oxford University Press, 2007), 130. See also Robin Findlay Hendry, "Emergence in Chemistry: Substance and Structure," in *The Routledge Handbook of Emergence,* ed. Sophie Gibb, Robin Findlay Hendry, and Tom Lancaster (New York: Routledge, 2021), 315–26.

[12]See Steven French, *The Structure of the World: Metaphysics and Representation* (Oxford, UK: Oxford University Press, 2014); Christopher Pincock, "A New Perspective on the Problem of Applying Mathematics," *Philosophia Mathematica* 12, no. 2 (2004): 135–61, https://doi.org/10.1093/philmat/12.2.135; and Christopher Pincock, *Mathematics and Scientific Representation* (Oxford, UK: Oxford University Press, 2012); Mauro Durato, *The Software of the Universe: An Introduction to the History and Philosophy of Laws of Nature* (Farnham, UK: Ashgate, 2005); and James Ladyman, "Structural Realism," in *The Stanford Encyclopedia of Philosophy*, Winter 2020 Edition, ed. Edward N. Zalta, https://plato.stanford.edu/archives/win2020/entries/structural-realism/.

[13]Patrick Suppes, *Introduction to Logic* (Princeton, NJ: Van Nostrand Reinhold, 1957); Patrick Suppes, "A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences," *Synthese* 12, no. 2-3 (1960): 287–301, https://doi.org/10.1007/BF00485107; and Frederick Suppe, *The Semantic Conception of Theories and Scientific Realism* (Chicago, IL: University of Illinois Press, 2000).

[14]Roman Frigg and James Nguyen, "Scientific Representation," in *The Stanford Encyclopedia of Philosophy,* Spring 2020 Edition, ed. Edward N. Zalta, https://plato.stanford.edu/archives/spr2020/entries/scientific-representation/.

[15]Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," *Communications on Pure and Applied Mathematics* 13, no. 1 (1960): 1–14. See also Mark Steiner, *The Applicability of Mathematics as a Philosophical Problem* (Cambridge, MA: Harvard University Press, 2002).

[16]Pincock, "A New Perspective on the Problem of Applying Mathematics," 135–61; and Pincock, *Mathematics and Scientific Representation*.

[17]John D. Barrow, *Theories of Everything: The Quest for Ultimate Explanation* (Oxford, UK: Oxford University Press, 1991), 11.

[18]Saunders Mac Lane, "Mathematical Models: A Sketch for the Philosophy of Mathematics," *American Mathematical Monthly* 88, no. 7 (1981): 463, https://doi.org/10.2307/2321751.

[19]Bas C. van Fraassen, *Scientific Representation: Paradoxes of Perspective* (Oxford, UK: Oxford University Press, 2008), 23.

[20]Otávio Bueno, "Models and Scientific Representations," in *New Waves in Philosophy of Science*, ed. P. D. Magnus and Jacob Busch (Basingstoke, UK: Palgrave Macmillan, 2010), 94, 95.

[21]Robin Findlay Hendry, "Structure as Abstraction," *Philosophy of Science* 83, no. 5 (2016): 1078, https://doi.org/10.1086/687939.

[22]Haskell B. Curry, *Foundations of Mathematical Logic* (New York: McGraw Hill, 1963); and Angelo Margaris, *First Order Mathematical Logic* (Waltham, MA: Blaisdell, 1967).

[23]David Hilbert, *Grundlagen der Geometrie 1902, The Foundations of Geometry*, trans. E. J. Townsend (Chicago, IL: Open Court, 1950); Alfred Tarski, "What Is Elementary Geometry?," *Studies in Logic and the Foundations of Mathematics* 27 (1959): 16–29, https://doi.org/10.1016/S0049-237X(09)70017-5; and Leon Henkin, Patrick Suppes, and Alfred Tarski, eds., *The Axiomatic Method: With Special Reference to Geometry and Physics: Proceedings of an International Symposium Held at the University of California, Berkeley, December 26, 1957–January 4, 1958* (Amsterdam, Netherlands: North-Holland, 1959), 16–29.

[24]David D. Reid, "Discrete Quantum Gravity and Causal Sets," *Canadian Journal of Physics* 79, no. 1 (2001): 1–16, https://doi.org/10.1139/p01-032; Fay Dowker, "Causal Sets as Discrete Spacetime," *Contemporary Physics* 47, no. 1 (2006): 5, https://doi.org/10.1080/17445760500356833; Carlo Rovelli, *Quantum Gravity* (New York: Cambridge University Press, 2009), 8, 9; and Christian Wüthrich, "The Structure of Causal Sets," *Journal for General Philosophy of Science* 43 (2012): 237–38, https://doi.org/10.1007/s10838-012-9205-1.

[25]Simo Knuuttila, "Time and Creation in Augustine," in *The Cambridge Companion to Augustine*, 2nd edition, ed. David Vincent Meconi and Eleonore Stump (New York: Cambridge University Press, 2014), 89.

[26]Medieval theologians include Thomas Bradwardine (1295–1339), Bonaventure (1221–1274), Thomas Aquinas (1225–1274), and Henry of Ghent (1217–1293). The reformers who affirmed it include Johannes Wollebius (1589–1629), *Abridgment of Christian Divinity*; Pedro Hurtado de Mendoza (1578–1641), *Universa philosophia* (Lyon 1624); Adriaan Heereboord (1614–1659), *Pneumatics* (1659); William Ames (1576–1633), *Marrow of Theology* (1648); Francis Turretin (1623–1687), *Institutio Theologiae Elencticae* (1688); Petrus van Mastricht (1630–1706), *Theoretico-Practica Theologia* (1699); Melchior Leydecker (1642–1721), *De providentia Dei* (1677), *Fax veritatis* (1677), and *Synopsis* (1684); and Johann Friedrich Stapfer (1708–1775), *Institutiones* (1743–1747).

[27]Genesis 18:14; Jeremiah 32:27; Matthew 19:26; Luke 1:37.

[28]Jonathan Edwards, *Concerning the End for Which God Created the World,* in *Ethical Writings*, vol. 8 of *The Works of Jonathan Edwards,* ed. Paul Ramsey (New Haven, CT: Yale University Press, 1989), 432.

[29]Edwards, *Concerning the End for Which God Created the World*, 413.

[30]These hold unless one is under moral obligation or subject to external standards of propriety and is acting purely out of respect for morality. These do not, however, apply to God.

[31]Two prominent exceptions are Norman Kretzmann, "A General Problem of Creation: Why Would God Create Anything at All?," in *Being and Goodness: The Concept of the Good in Metaphysics and Philosophical Theology,* ed. Scott MacDonald (Ithaca, NY: Cornell University Press, 1991), 208–28; and Philip J. Donnelly, "Saint Thomas and the Ultimate Purpose of Creation," *Theological Studies* 2, no. 1 (1941): 53–83, https://doi.org/10.1177/004056394100200104.

[32]Edwards, *Concerning the End for Which God Created the World*, 445, 450, 462, and 420.

[33]This applies even where the entity is part of a whole.

[34]Geerhardus Vos, *Reformed Dogmatics, Volume One: Theology Proper*, trans. and ed. Richard B. Gaffin Jr. (1896; reprint, Bellingham, WA: Lexham Press, 2012), 184.

[35]It is not *Berkeleyan idealism*. Rather, the universe is real with respect to creatures and involves genuine objects of scientific inquiry.

[36]Henry of Ghent (1217–1293) held that the difference between God's having a representation for a thing (*esse essentiae*) and God's volitionally conferring existence to the thing is an "intentional distinction" in God's mind. *Essence* and *being* are therefore different intentions, not different things. Porro explains Henry in this way: "Being is therefore an *intentio* that occurs to essence without adding anything real, and so it differs from essence only intentionally." Pasquale Porro, "Henry of Ghent," *The Stanford Encyclopedia of Philosophy,* Fall 2014 Edition, ed. Edward N. Zalta, 10, https://plato.stanford.edu/archives/fall2014/entries/henry-ghent/.

[37]Meinard Kuhlmann, "Processes as Objects of Quantum Field Theory: Consequences for the Interpretation of QFT," in *Things, Fact, and Events,* ed. Jan Faye, Uwe Scheffler, and Max Urchs (Amsterdam, Netherlands: Rodopi, 2000), 365–89; and Meinard Kuhlmann, "Quantum Field Theory," *The Stanford Encyclopedia of Philosophy,* Fall 2020 Edition, ed. Edward N. Zalta, https://plato.stanford.edu/archives/fall2020/entries/quantum-field-theory/.

[38]This is not *categorical* or *absolute* occasionalism. Like the occasionalism of Louis de La Forge (1632–1666) of the seventeenth century, this version differs from *pure*

occasionalism in that it applies *only* to physical causation, not to the intentions or choices of free agents.

[39]It is a matter of dispute among philosophers of logic whether truthbearers are *sentences*, *propositions*, or *judgments*. The view presented here shares features with a rough synthesis of the views of Friederike Moltmann, *Abstract Objects and the Semantics of Natural Language* (Oxford, UK: Oxford University Press, 2013); Jeffrey C. King, *The Nature and Structure of Content* (Oxford, UK: Oxford University Press, 2007); Scott Soames, *What Is Meaning?* (Princeton, NJ: Princeton University Press, 2010); C. B. Martin, *The Mind in Nature* (New York: Oxford University Press, 2010); Jeffrey C. King, Scott Soames, and Jeff Speaks, *New Thinking about Propositions* (Oxford, UK: Oxford University Press, 2014); and Eric Margolis and Stephen Laurence, "The Ontology of Concepts—Abstract Objects or Mental Representations?," *Noûs* 41, no. 4 (2007): 561–93, https://doi.org/10.1111/j.1468-0068.2007.00663.x.

[40]Friederike Moltmann, "Cognitive Products and the Semantics of Attitude Verbs and Deontic Modals," in *Act-Based Conceptions of Propositional Content*, ed. Friederike Moltmann and Mark Textor (Oxford, UK: Oxford University Press, 2017), 254–89.

[41]Christopher Menzel, "Theism, Platonism, and the Metaphysics of Mathematics," *Faith and Philosophy* 4, no. 4 (1987): 365–82, https://doi.org/10.5840/faithphil19874441; Christopher Menzel, "God and Mathematical Objects," in *Mathematics in a Postmodern Age*, ed. Russell W. Howell and W. James Bradley (Grand Rapids, MI: Eerdmans, 2001), 65–97; and Thomas V. Morris and Christopher Menzel, "Absolute Creation," *American Philosophical Quarterly* 23, no. 4 (1986): 353–62, https://www.jstor.org/stable/20014160.

[42]Paul Copan and William Lane Craig, *Creation out of Nothing: A Biblical, Philosophical, and Scientific Exploration* (Grand Rapids, MI: Baker Academic, 2004), 167–96.

[43]For arguments for this claim from differing angles, see Walter J. Schultz, "The Actual World from Platonism to Plans: An Emendation of Alvin Plantinga's Modal Realism," *Philosophia Christi* 16, no. 1 (2014): 81–100, https://www.academia.edu/en/10150447/The_Actual_World_from_Platonism_to_Plans_An_Emendation_of_Alvin_Plantinga_s_Modal_Realism; Walter J. Schultz, "Divine Action, Ontological Dependence, and Truthmaking," in *Mereologies, Ontologies and Facets: The Categorical Structure of Reality,* ed. Paul M. W. Hackett (Lanham, MD: Rowman & Littlefield Publishers, 2018), 201–33; and Walter J. Schultz and Lisanne D'Andrea Winslow, "The Structures of the Actual World," *Proceedings of the Association of Christians in the Mathematical Sciences* (2013): 168–86, https://acmsonline.org/home2/wp-content/uploads/2016/04/ACMS-2013-Proceedings.pdf.

[44]William P. Alston, *Realist Conception of Truth* (Ithaca, NY: Cornell University Press, 1996), 38.

[45]Ibid., 7, 8.

[46]Andrew Paul Ushenko, *The Problems of Logic* (Princeton, NJ: Princeton University Press, 1941), 121.

[47]Donald Davidson, "In Defense of Convention T," in *Truth, Syntax and Modality*, ed. Hugues Leblanc (Amsterdam, Netherlands: North-Holland, 1973), 76–86.

[48]This is one issue that divided Gottlob Frege and David Hilbert. Hilbert held that the consistency of axioms guaranteed the existence of the mathematical objects. Frege insisted that it cannot. Gottlob Frege, *Philosophical and Mathematical Correspondence,* ed. Gottfried Gabriel et al., trans. Hans Kaal (Chicago, IL: University of Chicago Press, 1980), 39, 40. See also Patricia Blanchette, "The Frege-Hilbert Controversy," *The Stanford Encyclopedia of Philosophy,* Fall 2018 Edition, ed. Edward N. Zalta, https://plato.stanford.edu/archives/fall2018/entries/frege-hilbert/.

[49]Mac Lane, *Mathematical Models,* 471.

[50]James Ladyman, "What Is Structural Realism?," *Studies in History and Philosophy of Science* 29, no. 3 (1998): 409–24; French, *The Structure of the World*; and Steven French and James Ladyman, "In Defense of Ontic Structural Realism," in *Scientific Structuralism, vol. 281: Boston Studies in the Philosophy of Science,* ed. Alisa Bokulich and Peter Bokulich (Heidelberg, Germany: Springer, 2011), 25–42.

[51]This conceptually coheres with Michael Esfeld, "The Modal Nature of Structures in Ontic Structural Realism," *International Studies in the Philosophy of Science* 23, no. 2 (2009): 179–94, http://philsci-archive.pitt.edu/id/eprint/4459. See also Wüthrich, "The Structure of Causal Sets."

[52]Kuhlmann, "Quantum Field Theory," 56.

[53]For an example, John P. Burgess, "Synthetic Mechanics," *Journal of Philosophical Logic* 13, no. 4 (1984): 379–95, https://www.jstor.org/stable/30226314; and John P. Burgess and Saul Kripke, "Synthetic Mechanics Revisited," *Journal of Philosophical Logic* 20, no. 2 (1991): 121–30, https://www.jstor.org/stable/30227246.

[54]Benacerraf, "Mathematical Truth," 666.

[55]Simply *being logically consistent* would make it true under a Platonist metaphysics.

[56]Although the three theses and the corollary constitute this ontological *account*, the account cannot serve as an *identifying account* of a proposition's truthmaker. In other words, the account cannot serve as a decision procedure or as an algorithm which takes the sentence as "input" and gives the truth-value of the proposition expressed by the sentence as "output."

[57]Shapiro, *Philosophy of Mathematics: Structure and Ontology*, 4.

[58]Benacerraf, "Mathematical Truth," 661.

[59]Ibid., 662.

[60]Shapiro, *Philosophy of Mathematics: Structure and Ontology*, 4.

[61]See Alvin Plantinga, *Warrant and Proper Function* (New York: Oxford University Press, 1993), 4, 194.

[62]Benacerraf, "Mathematical Truth," 667.

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